

The Color Singlet Relativistic Correction in J/ψ Photoproduction

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Abstract

The $O(v^2)$ relativistic correction for inelastic J/ψ photoproduction, in which heavy quark pairs are in the dominant Fock state of the quarkonium, is studied in the framework of NRQCD factorization. An assessment of its significance, particularly in comparison to the color octet contributions, is made. It is found that the impact on the energy distribution is negative in certain regions of phase space. The predictions are compared with photoproduction data from DESY-HERA.

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I. INTRODUCTION

Recent theoretical developments have been responsible for a resurgent interest in quarkonia physics: a factorized form for quarkonia production rates [6], based on the effective field theory, Non Relativistic QCD (NRQCD) [1], has been proposed. In this formalism, the inclusive production rates can be expressed as a function of the rates for creating pairs of free heavy quarks, $Q\bar{Q}$, in various configurations, and by a series of non-perturbative matrix elements. The former may be computed using perturbative QCD, while the matrix elements, which describe the transition of the $Q\bar{Q}$ into a hadron, are defined in NRQCD. Contrary to previous approaches, NRQCD factorization allows a systematic determination of relativistic corrections, which describe the effects of a heavy quark moving at a small velocity v in the rest frame of the quarkonium. The velocity scaling rules of NRQCD [15], which reveal the power law behavior with respect to v of the operators of the effective field theory, may be used to organize the matrix elements into a hierarchy. Although each quarkonium resonance is identified by the $Q\bar{Q}$ having a particular set of quantum numbers (such a $Q\bar{Q}$ state is the dominant Fock state of the quarkonium), the relativistic corrections to the production rates for that quarkonium, which accommodates the relative motion of the Q and \bar{Q} , may introduce a $Q\bar{Q}$ state with other quantum numbers. For example, the pair may be in a color octet configuration. The corresponding channels in the production rate, despite being at higher orders in v , can be phenomenologically significant in certain kinematic domains, due to the particulars of the production process.

One of the experiments used to test NRQCD factorization is photoproduction of quarkonia, in which a real photon collides with a nucleon. Charmonium photoproduction has been investigated in the framework of the color singlet model [8], and more recently using NRQCD factorization [9], whose predictions have been subject to a comparison with preliminary data from HERA [2,4]. In addition to the channel in which the $Q\bar{Q}$ is in the dominant Fock state of the J/ψ , corrections, corresponding to the $Q\bar{Q}$ in spin singlet and spin triplet, S and P wave, color octet states, have been considered in the studies. The numerical values of the color octet matrix elements, were estimated in an earlier work [7], by fitting theoretical predictions for prompt charmonium production in $p\bar{p}$ processes, to data from the Fermilab-Tevatron. Using these values, however, leads to a significant discrepancy with the photoproduction rates at HERA: the contributions from octet channels overwhelm the data in certain regions of phase space.

A number of uncertainties which enter into calculations may be responsible for the inconsistency between the HERA and Tevatron data for charmonium production, when viewed from the perspective of NRQCD. Among these are uncertainties in parton distribution functions, the value of the charm quark mass, higher twist effects [11], and higher order perturbative QCD corrections. In the work by Kniehl [20], important higher order perturbative QCD effects, due to initial state multiple gluon radiation, were included when analyzing hadroproduction at the Tevatron. The re-estimated matrix elements have values significantly smaller than the originally determined ones. Correspondingly, the HERA data is in much better agreement with the NRQCD predictions. This is true throughout most of phase space, except near certain boundaries where the octet contributions diverge. The authors in [12], have suggested this is due to the non-relativistic expansion breaking down in such regions. They have proposed a resummation of the non-relativistic expansion, which requires

the introduction of universal structure functions.

The contributions to J/ψ photoproduction from the color octet channels are at order v^4 in the small velocity expansion, relative to the leading order, color singlet term. There is a relativistic correction at order v^2 , incorporating the effects of the relative motion of the heavy quark pair in the dominant Fock state of J/ψ . The purpose of this paper is to examine this correction in the framework of NRQCD factorization, which requires the introduction of an additional matrix element. Previously, models have been constructed to describe such an order v^2 effect. The one parameter model presented in [18] has been used to examine quarkonia photoproduction by Jung et al. [13]; a two parameter model has also been proposed [14]. The $O(v^2)$ contribution has not been computed within NRQCD. It has been pointed out by Bodwin et al. [6], that corrections of $O(v^2)$ may be as important as radiative corrections of $O(\alpha_s)$, which have already been calculated for photoproduction [10], and found to be significant. Without a detailed understanding of the $O(v^2)$ term, it is not possible to make an assessment of the significance of the color octet terms: the $O(v^2)$ contribution is clearly necessary for consistency, according to the power counting rules of NRQCD.

The remainder of the paper is organized as follows. In the next section, some details of the calculation of the relativistic correction is described. In section III, numerical results, indicating the impact of the v^2 correction, are presented, together with a comparison with data from HERA. Section IV contains some concluding remarks.

II. PHOTOPRODUCTION

NRQCD factorization allows the inclusive cross section for the photoproduction of J/ψ to be written in the following form:

$$\frac{d\hat{\sigma}}{d\hat{t}}_{\gamma+i \rightarrow J/\psi+X} = \sum_n \frac{F_n}{m^{d_n-4}} \langle \mathcal{O}_n^{J/\psi} \rangle. \quad (1)$$

The coefficients, F_n depend on the kinematic invariants of the process in which a photon γ and a parton i from the initial hadron, react to produce a $Q\bar{Q}$. Here m is the mass of heavy quark. The production is a short distance process, and is therefore calculable using QCD perturbation theory. The subscript $n = {}^{2S+1}L_J^{(1,8)}$ specifies the spin S , orbital angular momentum L , and the total angular momentum J , of the $Q\bar{Q}$. The 1 and 8 means that the pair may be in a color singlet or color octet state. The matrix elements $\langle \mathcal{O}_n^{J/\psi} \rangle$, of mass dimension d_n describes the transition of the $Q\bar{Q}$ in the state n , into a J/ψ . Although J/ψ is identified as a $Q\bar{Q}$ mainly in a ${}^3S_1^{(1)}$ configuration, the short distance process may produce a heavy quark pair which is not in a ${}^3S_1^{(1)}$ state. The formation of the bound state can involve the emission of soft partons, which return the pair into the appropriate spectroscopic configuration. The relative importance of various terms in the sum in Eq. (1), may be determined by how each term is suppressed by powers of the strong coupling constant, α_s , whose dependence lies in F_n , and the velocity of the heavy quark in the rest frame of the quarkonium v , appearing in $\langle \mathcal{O}_n^{J/\psi} \rangle$. For J/ψ , $v^2 \approx 0.3$.

The dominant short distance contribution to inelastic photoproduction of J/ψ is the photon-gluon fusion process

$$\gamma(k) + G(g_1) \rightarrow Q\bar{Q}_n(P) + G(g_2). \quad (2)$$

The four momenta are specified in parenthesis. The process (2) at leading order in QCD perturbation theory is represented in Figs. 1(a) and 1(b). Both diagrams allow the heavy quark pair to be produced in a color octet state, while the $Q\bar{Q}$ can only be in a color singlet state in Fig. 1(a). Subprocesses involving light quarks in the initial state are considerably suppressed at the energies available at HERA [9]. Resolved processes, in which the photon breaks up into partons, which subsequently interacts with the nucleon, have been analyzed within NRQCD factorization [19]. These are important in the kinematic regime $z \leq 0.3$, where $z = E_{J/\psi}/E_\gamma$, in which the energies are defined in the nucleons rest frame.

The formation of J/ψ from a $Q\bar{Q}$ at leading order in the small velocity expansion is described by the matrix element

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{(1)}) \rangle = \left\langle 0 \left| \chi^\dagger \sigma^i \psi \left(\sum_{S, J_z} |J/\psi + S\rangle \langle J/\psi + S| \right) \psi^\dagger \sigma^i \chi \right| 0 \right\rangle. \quad (3)$$

The field ψ annihilates a heavy quark, while χ creates a heavy anti-quark. The matrix element is proportional to the rate of production of a J/ψ from the NRQCD vacuum, via an intermediate state in which the heavy quark pair is in a $^3S_1^{(1)}$ configuration. S represents light hadrons in the final state, and J_z is the helicity of the J/ψ , both of which are summed over.

The relativistic correction, appearing at $O(v^2)$ relative to the leading contribution, in which the heavy quark pair is also in the dominant Fock state of the J/ψ , involves the matrix element

$$\langle \mathcal{P}^{J/\psi}(^3S_1^{(1)}) \rangle = \left\langle 0 \left| \frac{1}{2} \left[\chi^\dagger \sigma^i \psi \left(\sum_S |J/\psi + S\rangle \langle J/\psi + S| \right) \psi^\dagger \sigma^i \left(-\frac{i}{2} \vec{\mathbf{D}} \right)^2 \chi + h.c. \right] \right| 0 \right\rangle, \quad (4)$$

where $\chi^\dagger \vec{\mathbf{D}} \psi = \chi^\dagger(\mathbf{D})\psi - (\mathbf{D}\chi)^\dagger\psi$. Therefore, the differential cross section to $O(v^2)$ takes the form,

$$\frac{d\hat{\sigma}}{d\hat{t}}_{\gamma+G \rightarrow J/\psi+X} = \frac{F(^3S_1^{(1)})}{m^2} \langle \mathcal{O}^{J/\psi}(^3S_1^{(1)}) \rangle + \frac{G(^3S_1^{(1)})}{m^4} \langle \mathcal{P}^{J/\psi}(^3S_1^{(1)}) \rangle + O(v^4). \quad (5)$$

Of the octet contributions which appear at $O(v^4)$, the $^1S_0^{(8)}$ and $^3P_J^{(8)}$ channels were found to be the most important. The structure of the matrix elements which describe these transitions, are similar to Eqs.(3) and (4), except for the operators between the quark and anti-quark fields; they may be found in the literature [6]. The aim in this section is to determine the coefficient $G(^3S_1^{(1)})$, containing the short distance information associated with the v^2 correction.

The low energy equivalence of full QCD and NRQCD allow the short distance coefficients F_n to be determined by the following matching condition:

$$\frac{d\hat{\sigma}}{d\hat{t}}_{\gamma+G \rightarrow Q\bar{Q}_n+X} \Big|_{pQCD} = \sum_n \frac{F_n}{m^{d_n-4}} \langle 0 | \mathcal{O}_n^{Q\bar{Q}} | 0 \rangle \Big|_{pNRQCD}. \quad (6)$$

On the left hand side of Eq. 6), the cross section for the production of a free $Q\bar{Q}$ is computed using perturbative QCD. The matrix elements on the right hand side are evaluated using

perturbative NRQCD. Both sides are expanded in powers of \mathbf{q} , the momentum of the heavy quark in the rest frame of the quarkonium. The F_n are then found by comparing powers of \mathbf{q} . The QCD side of the matching condition involves expanding the amplitude for the process, Eq. (2), in powers of \mathbf{q} . The leading terms contain $Q\bar{Q}$'s in the dominant Fock state of the J/ψ , ${}^3S_1^{(1)}$, as well as some octet configurations: ${}^3S_1^{(8)}$ and ${}^1S_0^{(8)}$. Terms at order \mathbf{q} in the amplitude project out heavy quark pairs in ${}^3P_J^{(8)}$ states, but do not contain a heavy quark pair in ${}^3S_1^{(1)}$ states. In order to determine the $O(v^2)$ contribution in (5), it is necessary to expand the QCD amplitude to $O(\mathbf{q}^2)$: at this order, heavy quark pairs in ${}^3S_1^{(1)}$ configurations are present. The reason that the color octet contributions appear at a higher order in v , in (5), despite being at a lower order in the perturbative amplitude, is because the power counting is different for hadronic states: additional v suppression appears in the hadronic projection operators of the color octet matrix elements.

Some conventions used in [16] will be adopted in this work. The four momenta of the heavy quark and anti-quark is

$$\begin{aligned} p^\mu &= \frac{P^\mu}{2} + \Lambda_i^\mu q^i, \\ \bar{p}^\mu &= \frac{P^\mu}{2} - \Lambda_i^\mu q^i, \end{aligned} \quad (7)$$

where Λ is the Lorenz boost matrix from the rest frame of the J/ψ , to the frame in which it is moving with four momentum $P = (\sqrt{4(m^2 + |\mathbf{q}|^2)} + \mathbf{P}^2, \mathbf{P})$. The partonic Mandelstam invariants are defined in the standard manner: $\hat{s} = (k + g_1)^2 = (P + g_2)^2$, $\hat{t} = (k - P)^2 = (g_1 - g_2)^2$ and $\hat{u} = (g_1 - P)^2 = (k - g_2)^2$. Only two of these invariants are independent, their relationship is given by

$$\hat{s} + \hat{t} + \hat{u} = P^2 = 4(m^2 + |\mathbf{q}|^2). \quad (8)$$

We choose \hat{u} to be expressed in terms of the other two invariants, and $|\mathbf{q}|^2$, which is associated with $\langle \mathcal{P}^{J/\psi}({}^3S_1^{(1)}) \rangle$. Two contraction identities found in reference [16] will be frequently used in the calculation:

$$\begin{aligned} g_{\mu\nu} \Lambda_i^\mu \Lambda_j^\nu &= -\delta^{ij}, \\ \Lambda_i^\mu \Lambda_i^\nu &= -g_{\mu\nu} + \frac{P^\mu P^\nu}{P^2}. \end{aligned} \quad (9)$$

First we will consider the QCD side of Eq. (6). The transition matrix for the process may be written,

$$\mathcal{T}_{\gamma+G \rightarrow Q\bar{Q}_n+G} = \varepsilon_\mu(k) \varepsilon_\alpha^a(g_1) \varepsilon_\beta^{*b}(g_2) \bar{u}(p) T_{ab}^{\mu\alpha\beta} v(\bar{p}).$$

The color indices of the gluons are specified by a and b . Following the techniques used in [16], $T_{ab}^{\mu\alpha\beta}$ is decomposed in terms of the sixteen basis matrices $\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5$, and $\sigma_{\mu\nu}$. When these are inserted between the quark and anti-quark spinors $\bar{u}(p)$ and $v(\bar{p})$, \mathcal{T} may be expressed in terms of the color triplet, Pauli spinors, ξ and η . Projecting out the color singlet channel and expanding to $O(|\mathbf{q}|^3)$, the transition matrix takes the form

$$\mathcal{T} = \varepsilon_\mu(k) \varepsilon_\alpha^a(g_1) \varepsilon_\beta^{*b}(g_2) \frac{\delta_{ab}}{2N_c} (a_i^{\mu\alpha\beta} + b_{imn}^{\mu\alpha\beta} q^m q^n) \xi^\dagger \sigma^i \eta + \dots \quad (10)$$

The terms $a_i^{\mu\alpha\beta}$ and $b_{imn}^{\mu\alpha\beta}$ are functions of the kinematic variables and the Lorenz boost matrices. The ellipsis represents contributions in which the $Q\bar{Q}$ pairs are in color singlet P wave states, spin singlet states, or terms which contribute to processes occurring at $O(v^3)$ or higher. To obtain the rate, \mathcal{T} is multiplied by its complex conjugate, and the appropriate sums and averages over spin and color degrees of freedom are made. Before the momentum space integration is carried out, a change of variables is performed: $(p, \bar{p}) \rightarrow (P, q)$, to facilitate the matching as defined in Eq. (6). The perturbative cross section becomes

$$\frac{d\hat{\sigma}}{d\hat{t}}_{\gamma+G \rightarrow Q\bar{Q}n+X} = -\frac{1}{16\pi\hat{s}^2} \left(\frac{1}{2^2 \cdot 4N_c^2} \right) \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[A_{ij} + \left(b_{ij}^{(2)} - \frac{A_{ij}}{2m^2} \right) |\mathbf{q}|^2 + c_{ijmn}^{(2)} q^m q^n \right] \sigma^i \otimes \sigma^j + \dots \quad (11)$$

For convenience, the notation suppressing the Pauli spinors and the sums over the spins has been employed [6]. The coefficients appearing in the transition matrix are related to the ones appearing in (11). Using the identities in (9), the contraction of the Lorenz indices relates the A_{ij} , $b_{ij}^{(2)}$ and $c_{ijmn}^{(2)}$, to $a_i^{\mu\alpha\beta}$ and $b_{imn}^{\mu\alpha\beta}$.

$$\begin{aligned} a_i^{\mu\alpha\beta} a_{\mu\alpha\beta,j}^* &= A_{ij} + b_{ij}^{(2)} |\mathbf{q}|^2 + O(|\mathbf{q}|^3) \\ a_i^{\mu\alpha\beta} b_{\mu\alpha\beta,imn}^* + b_{imn}^{\mu\alpha\beta} a_{\mu\alpha\beta,j}^* &= c_{ijmn}^{(2)} + O(|\mathbf{q}|) \end{aligned}$$

It must be kept in mind that the factor A_{ij} implicitly retains some \mathbf{q} dependence which will become fully exposed once contractions over the remaining indices are carried out. The term $A_{ij}/2m^2$ is a consequence of the change of variables. Using rotational symmetry, Eq. (11) takes the form

$$\begin{aligned} \frac{d\hat{\sigma}}{d\hat{t}}_{\gamma+G \rightarrow Q\bar{Q}n+X} &= -\frac{1}{16\pi\hat{s}^2} \left(\frac{1}{2^2 \cdot 4N_c^2} \right) \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[\frac{1}{3} c_{ii}^{(0)} \sigma^j \otimes \sigma^j + \frac{1}{3} \left(3c_{ii}^{(2)} + b_{ii}^{(2)} - \frac{c_{ii}^{(0)}}{2m^2} \right) \right. \\ &\quad \left. \sigma^j \otimes \sigma^j |\mathbf{q}|^2 + \frac{1}{30} \left[(4c_{iijj}^{(2)} - c_{ijij}^{(2)} - c_{ijji}^{(2)}) |\mathbf{q}|^2 \sigma^m \otimes \sigma^m + (-2c_{iijj}^{(2)} + 3c_{ijij}^{(2)} + 3c_{ijji}^{(2)}) \mathbf{q} \cdot \sigma \otimes \mathbf{q} \cdot \sigma \right] \right]. \end{aligned} \quad (12)$$

Only the leading order terms in $|\mathbf{q}|^2$ have been kept in the contractions of $c_{ijmn}^{(2)}$ and $b_{ij}^{(2)}$. $c_{ii}^{(0)}$ and $c_{ii}^{(2)}$ are defined to be

$$A_{ii} = c_{ii}^{(0)} + c_{ii}^{(2)} |\mathbf{q}|^2.$$

Equation (12) displays all the \mathbf{q}^2 dependence necessary to determine $G(^3S_1^{(1)})$. The coefficients have been expressed in terms of the invariants \hat{s} and \hat{t} using the contraction identities. They are somewhat lengthy and will not be presented here.

Next the NRQCD side of the matching condition is considered. The $Q\bar{Q}$ differential cross section, only including the contributions we are interested in, may be written

$$\begin{aligned} &\frac{F(^3S_1^{(1)})}{m^2} \langle \mathcal{O}^{Q\bar{Q}}(^3S_1^{(1)}) \rangle + \frac{G(^3S_1^{(1)})}{m^4} \langle \mathcal{P}^{Q\bar{Q}}(^3S_1^{(1)}) \rangle + \\ &\frac{F(^3S_1^{(1)}, {}^3D_1^{(1)})}{m^4} \langle \mathcal{P}^{Q\bar{Q}}(^3S_1^{(1)}, {}^3D_1^{(1)}) \rangle + \dots \end{aligned} \quad (13)$$

It is necessary to take into account another color singlet matrix element with $d_n = 8$,

$$\langle \mathcal{P}^{Q\bar{Q}}(^3S_1^{(1)}, ^3D_1^{(1)}) \rangle = \left\langle 0 \left| \frac{1}{2} \left[\chi^\dagger \sigma^i \psi \left(\sum_S |Q\bar{Q}\rangle \langle Q\bar{Q}| \right) \psi^\dagger \sigma^j \left(-\frac{i}{2} \right)^2 \overset{\leftrightarrow}{D}^{(i \leftrightarrow j)} \chi + h.c \right] \right| 0 \right\rangle. \quad (14)$$

The symmetric traceless tensor, formed from the covariant derivatives is specified by $\overset{\leftrightarrow}{D}^{(i \leftrightarrow j)} = (\overset{\leftrightarrow}{D}^{i \leftrightarrow j} + \overset{\leftrightarrow}{D}^{j \leftrightarrow i})/2 - \overset{\leftrightarrow}{D} \cdot \overset{\leftrightarrow}{D} / 3$. Here $\langle \mathcal{P}^{Q\bar{Q}}(^3S_1, ^3D_1) \rangle$ describes the overlap of the transition amplitudes in which the $Q\bar{Q}$ is in spin triplet S and D wave states. Therefore, $F(^3S_1, ^3D_1)$ mixes with $F(^3S_1)$ in perturbation theory. $\langle \mathcal{P}^{Q\bar{Q}}(^3S_1, ^3D_1) \rangle$ appears at $O(v^2)$ in perturbation theory, where quarks and gluons are free. However, because the velocity counting is different for hadronic states, the corresponding hadronic matrix element $\langle \mathcal{P}^{J/\psi}(^3S_1, ^3D_1) \rangle$ is at order v^4 . This is because it requires two chromo-electric dipole transitions to take a $Q\bar{Q}$ in a $^3S_1^{(1)}$ configuration, to a $^3D_1^{(1)}$ state, costing a factor of v^2 in the projection operator of $\langle \mathcal{P}^{J/\psi}(^3S_1^{(1)}, ^3D_1^{(1)}) \rangle$. The covariant derivatives in the operator contributes another v^2 .

Using a non-relativistic normalization convention, it is possible to realize the right hand side of (6) at leading order in perturbative NRQCD:

$$\begin{aligned} & \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \left[\frac{F(^3S_1^{(1)})}{m^2} \sigma^i \otimes \sigma^j + \frac{1}{3m^4} (3F(^3S_1^{(1)}, ^3D_1^{(1)})) \mathbf{q} \cdot \sigma \otimes \mathbf{q} \cdot \sigma \right. \\ & \left. + 3 [G(^3S_1^{(1)}) - F(^3S_1^{(1)}, ^3D_1^{(1)})] \mathbf{q}^2 \sigma^i \otimes \sigma^i \right]. \end{aligned} \quad (15)$$

One may worry about the use of different normalization of states on either side of (6). However this is not a problem as only physical quantities are being considered, and the Pauli spinors, which are not shown explicitly, have the same normalizations: $\xi^\dagger \xi = 1$ and $\eta^\dagger \eta = 1$. Braaten et al. [16] defined the matrix elements with relativistic normalizations, which were subsequently related to the matrix elements defined in NRQCD.

By comparing Eq. (15) with Eq. (12), we finally obtain the short distance coefficients. The leading order color singlet contribution is well known:

$$\frac{F(^3S_1^{(1)})}{m^2} = \frac{1}{16\pi\hat{s}^2} \frac{32m}{27} (4\pi\alpha_s)^2 (4\pi\alpha) e_c^2 \cdot \frac{\hat{s}^2(\hat{s} - 4m^2)^2 + \hat{t}^2(\hat{t} - 4m^2)^2 + (4m^2 - \hat{s} - \hat{t})^2(\hat{s} + \hat{t})^2}{(\hat{s} - 4m^2)^2(\hat{t} - 4m^2)^2(\hat{s} + \hat{t})^2}. \quad (16)$$

The strong coupling constant and the fine structure constant are given by α_s and α . The electromagnetic charge of the charm quark in units of e , is e_c . As a consequence of Eq. (8), Eq. (16) is expressed in terms of the combination $4m^2 - \hat{s} - \hat{t}$, instead of \hat{u} . The perturbative coefficient of the relativistic correction for the color singlet channel is found to be

$$\begin{aligned} \frac{G(^3S_1^{(1)})}{m^4} = & \frac{1}{16\pi\hat{s}^2} 32(4\pi\alpha)^2 (4\pi\alpha) e_c^2 (-6144m^{10}\hat{s}^2 + 1280m^8\hat{s}^3 + 960m^6\hat{s}^4 \\ & - 336m^4\hat{s}^5 + 28m^2\hat{s}^6 - 4096m^{10}\hat{s}\hat{t} - 512m^8\hat{s}^2\hat{t} + 3200m^6\hat{s}^3\hat{t} \\ & - 1008m^4\hat{s}^4\hat{t} + 72m^2\hat{s}^5\hat{t} + \hat{s}^6\hat{t} - 6144m^{10}\hat{t}^2 - 512m^8\hat{s}\hat{t}^2 + 3200m^6\hat{s}^2\hat{t}^2 \\ & - 1408m^4\hat{s}^3\hat{t}^2 + 92m^2\hat{s}^4\hat{t}^2 + 3\hat{s}^5\hat{t}^2 + 1280m^8\hat{t}^3 + 3200m^6\hat{s}\hat{t}^3) \end{aligned}$$

$$\begin{aligned}
& - 1408m^4\hat{s}^2\hat{t}^3 + 112m^2\hat{s}^3\hat{t}^3 + 5\hat{s}^4\hat{t}^3 + 960m^6\hat{t}^4 - 1008m^4\hat{s}\hat{t}^4 \\
& + 92m^2\hat{s}^2\hat{t}^4 + 5\hat{s}^3\hat{t}^4 - 336m^4\hat{t}^5 + 72m^2\hat{s}\hat{t}^5 + 3\hat{s}^2\hat{t}^5 + 28m^2\hat{t}^6 + \hat{s}\hat{t}^6) / \\
& (81m(\hat{s} - 4m^2)^3(\hat{t} - 4m^2)^3(\hat{s} + \hat{t})^3).
\end{aligned} \tag{17}$$

This result is different to the one found in [13]. First, the relativistic correction in this work is expressed in terms of a matrix element defined in NRQCD; in [13], it is correspondingly a parameter ϵ , defined via

$$M_{J/\psi} = 2m + \epsilon, \tag{18}$$

where ϵ was taken to be positive. As pointed out in [11], this parameter can be negative and can be defined as a matrix element in NRQCD which is different to the one appearing in Eq. (5). Secondly, the correction calculated in [13], is based on the model proposed in [18], in which the contribution of the momentum \mathbf{q} to the energy of the heavy quark is neglected—that is the energy is taken to be $M_{J/\psi}/2$, in the quarkonium rest frame; it really should be $\sqrt{m^2 + \mathbf{q}^2}$. This implies that our result cannot be obtained from that presented in [13] by setting $M_{J/\psi} = 2m$, and the parameter ϵ proportional to $\langle \mathcal{P}^{J/\psi}(^3S_1^{(1)}) \rangle$.

III. NUMERICAL RESULTS

In this section an assessment of the significance of the order v^2 correction shall be made, using the analytic result (17). The predictions will be compared to photoproduction data from HERA. In [9], the color octet channels $^1S_0^{(8)}$ and $^3P_J^{(8)}$ were found to be important in photoproduction of J/ψ . The corresponding matrix elements were taken to have numerical values consistent with the linear combination [7]

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)}) \rangle + \frac{3}{m^2} \langle \mathcal{O}^{J/\psi}(^3P_0^{(8)}) \rangle = 6.6 \times 10^{-2} \text{ GeV}^3, \tag{19}$$

determined in the original analysis of hadroproduction rates at the Tevatron. However, these values lead to predictions which are significantly greater than the experimental results. The inclusion of effects due to initial state gluon radiation modifies the linear combination of the matrix elements thus [20]:

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)}) \rangle + \frac{3.54}{m^2} \langle \mathcal{O}^{J/\psi}(^3P_0^{(8)}) \rangle = 5.27 \times 10^{-3} \text{ GeV}^3. \tag{20}$$

In this study, we shall assume the latter, and take $\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)}) \rangle = \langle \mathcal{O}^{J/\psi}(^3P_0^{(8)}) \rangle / m^2 = 1.2 \times 10^{-3} \text{ GeV}^3$. The leading order color singlet matrix element is taken to have the value:

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{(1)}) \rangle = 1.3 \text{ GeV}^3. \tag{21}$$

A naive estimate for $\langle \mathcal{P}^{J/\psi}(^3S_1^{(1)}) \rangle$ shall be used, based on the power law suppression of $\langle \mathcal{P}^{J/\psi}(^3S_1^{(1)}) \rangle$ relative to $\langle \mathcal{O}^{J/\psi}(^3S_1^{(1)}) \rangle$:

$$\langle \mathcal{P}^{J/\psi}(^3S_1^{(1)}) \rangle = 0.85 \text{ GeV}^5. \tag{22}$$

The other inputs for the computations are as follows: the strong coupling constant at the charm mass scale is taken to be $\alpha_s(m) = 0.3$, with $m = 1.48$ GeV. The MRSA determination of the gluon structure function is used [17], with $\Lambda_{QCD}^{\overline{MS}, n_f=4} = 300$ MeV, and $Q^2 = (2m)^2$.

The total cross section as a function of the center of mass energy is considered first. The constraints: $z \leq 0.8$ and $p_t \geq 1$ GeV, valid at HERA, are adopted. The K factor, consistent with these cuts, from the NLO corrections to the color singlet channel [10] is $K \approx 1.8$. Figure 2 shows the total cross section as a function of the photon-nucleon center of mass energy, \sqrt{s} . The leading order color singlet contribution (long dashed line), accounts quite well for most of the data from H1 [3] (circles) and ZEUS [4] (squares). The v^2 correction (short dashed line), together with the color octet terms (dot-dash line), contributes only marginally. The octet contributions are greater than the $O(v^2)$ correction by approximately a factor of four, for large center of mass energies, with the values of the matrix elements that have been adopted. If the octet matrix elements were chosen to be consistent with (19), then the v^2 correction would be smaller by approximately a factor of ten, relative to the octet contributions. However, $\langle \mathcal{O}^{J/\psi}(^3S_1^{(1)}) \rangle$ may be somewhat larger than 0.85 GeV⁵, therefore both corrections may have similar magnitudes.

Next, the $d\sigma/dz$ distribution is considered, as a function of z . Figure 3(a) shows the ratio of the v^2 to the octet contributions, at $\sqrt{s} = 100$ GeV (solid line) and $\sqrt{s} = 14.7$ GeV (dashed line). At $\sqrt{s} = 100$ GeV the cut, $p_t \geq 1$ GeV, consistent with results from HERA is adopted, while at $\sqrt{s} = 14.7$, $p_t \geq 0.1$ GeV, valid for the European Muon Collaboration (EMC). For $\sqrt{s} = 100$ GeV, it can be seen that the v^2 channel dominates at low z . For $z > 0.63$ the ratio is negative, indicating that the v^2 distribution is negative in that region. At $z \approx 0.7$ the ratio has a minimum value of about 0.45, meaning that in this region, the v^2 contribution is significant with respect to the color octet channels. Again, if the original determinations of the octet matrix elements were adopted, the v^2 contributions will give a negligible result at intermediate and large values of z , while at low z , they would be comparable to the color octet contributions. At the EMC energy regime, $\sqrt{s} = 14.7$ GeV, v^2 corrections are relatively more substantial at low z , while at larger values, not as important. Figure 3(b) is the z distribution at the center of mass energy $\sqrt{s} = 100$ GeV. The long dashed line represents the leading order contribution. The sum of the leading order and v^2 correction are shown with a dot-dash; the sum of the leading order and octet contributions is shown with the dashed line. The net result is shown with a solid line. The circles represent data from the H1 Collaboration [3]. The v^2 correction tends to decrease the distribution for larger values of z , while at smaller values, it is enhanced. Figure 3(c) shows the distribution at $\sqrt{s} = 14.7$ GeV, displaying a similar behavior, although the v^2 effect is not as pronounced. The data is from the EMC [5]. It should be noted that the large z region where the v^2 contribution is most substantial, is also where the non relativistic expansion breaks down [12]. Our analysis, leads to a qualitatively different behavior for the energy distribution, than in the work by Jung et al. [13]. In their case there is an enhancement of $d\sigma/dz$ for large values of z .

The transverse momentum distribution for photoproduction was also examined. However, it was found that the v^2 correction leads to a negligible contribution to the overall differential rate. It is expected that a similar v^2 correction appears in $p + \bar{p} \rightarrow J/\psi + X$ processes at the Tevatron, where color octet fragmentation channels dominate at large transverse momentum. The v^2 correction can be obtained by realizing that it originates via a

process similar to that depicted in figure 1(a), except with the initial photon replaced by a gluon. The color singlet rates, including the v^2 correction is obtained by multiplying $F(^3S_1^{(1)})$ and $G(^3S_1^{(1)})$ by the factor $5\alpha_s/(96\alpha e_c^2)$, which takes into account the differences in the initial state. However it was found that the v^2 correction contributes insignificantly to the transverse momentum distribution for moderate transverse momenta, at the Tevatron center of mass energy $\sqrt{s} = 1.8$ TeV.

IV. CONCLUSION

In this work, the relativistic correction to the color singlet channel in J/ψ photoproduction was calculated using the NRQCD factorization formalism. The correction is described by an additional matrix element which appears at $O(v^2)$ relative to the leading order color singlet term, according to the velocity scaling rules of NRQCD. This work contrasts with previous studies in that it draws qualitatively different conclusions about the behavior of observable. Specifically, the contribution leads to a decrease in the energy distribution for large values of z , while for smaller values there is an enhancement of the differential cross section. The effect of the correction in photoproduction seems to be more significant for larger center of mass energies. In some regions of phase space, the v^2 correction is comparable to the color octet contributions, with the values of the parameters that were used. However, it has been noted earlier that the rates are sensitive to $\alpha_s(m)$, m , and the choice of parton distribution functions, which are not precisely known. Certainly it seems that the uncertainty in these parameters, correspond to variations in the rates which are of similar magnitude, or greater than the effect of the $O(v^2)$ correction [10]. Therefore, at the present level of precision, it is not possible to observe the effect of the correction in experiments.

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REFERENCES

- [1] W.E. Caswell and G.P. Lepage, Phys. Lett. B **167**, 437 (1986).
- [2] H1 Collab., S. Aid et al., Nucl. Phys. B **472**, 3 (1996).
- [3] B. Naroska (H1 collab.), Private Communications (1999).
- [4] ZEUS Collab., J. Breitweg et al., Z. Phys. C **76**, 599 (1997).
- [5] EMC Collab., J. Ashman et al. , Z. Phys. C **56** 21.
- [6] G.T Bodwin, E. Braaten, and, G.P Lepage, Phys. Rev. D **51** 1125 (1995).
- [7] P. Cho and A.K Leibovich,Phys. Rev. D **53**, 150 (1995).
P. Cho and A.K Leibovich,Phys. Rev. D **53**, 6203 (1995).
- [8] E. L. Berger and D. Jones, Phys. Rev. D **23**, 1521 (1981).
- [9] P. Ko, J. Lee and H.S Song, Phys. Rev. D **54** 4312 (1996).
M. Cacciari and M. Krämer, Phys. Rev. Lett. **76** 4128 (1996).
- [10] M. Krämer, J. Zunft, J. Steegbirk and P.M. Zerwas, Phys. Lett. B**348**, 657 (1995).
M. Krämer, Nucl. Phys. **B459**, 3 (1996).
- [11] J.P. Ma, Nucl. Phys. **B498**, 267 (1997).
- [12] M. Beneke, I.Z Rothstein and M.B. Wise, Phys. Lett. **B408**, 373 (1997).
- [13] H. Jung, D. Krücker, C. Greub and D. Wyler, Z. Phys. C **60**, 721 (1993).
- [14] H. Khan and P. Hoodbhoy,Phys. Lett. B**382** 189 (1996).
- [15] G. P. Lepage, L. Magnea, C. Nakhleh, C. Magnea and K. Hornbostel, Phys. Rev. D **36**, 4052 (1992).
- [16] E. Braaten and Y. Chen, Phys.Rev. D **54** (1996) 3216.
S. Fleming and I. Maksymyk, Phys.Rev. D **54**, 3608 (1996).
- [17] A. D. Martin, R. G. Roberts, W. J. Stirling and R.S. Thorne, Eur. Phys. J. **C4**,463 (1998).
- [18] W.-Y. Keung and I.J. Muzinich, Phys. Rev. D**27** (1983) 1518.
- [19] M. Cacciari and M. Krämer, Contribution to the proceedings of the workshop ‘Future Physics at HERA’.
- [20] B. A. Kniehl, To appear in the proceedings of the workshop on Monte Carlo Generators for HERA Physics, DESY, Hamburg, 21 April 1998 - 5 February 1999, edited by T. Doyle, G. Grindhammer, G. Ingelman, and H. Jung.

Figure Captions

Figure 1: (a) One of six Feynman diagrams for the production of a $Q\bar{Q}$ pair. The others are obtained through permutations of photon and gluon lines. The $Q\bar{Q}$ may be in a color singlet or color octet configuration. (b) A typical diagram in which the $Q\bar{Q}$'s are only in color octet states.

Figure 2: The total cross section $\sigma(\sqrt{s})$ (nb) as a function of the center of mass energy \sqrt{s} (GeV). The leading order color singlet contribution is shown with a long dashed line. The $O(v^2)$ correction is the dot-dash line; the octet contribution is the short dash line. The total rate is the solid line.

Figure 3: (a) $d\sigma/dz$: the ratio of the v^2 contribution to the octet contributions as a function of z . The solid line is at $\sqrt{s} = 100$ GeV, and the dashed line is at $\sqrt{s} = 14.7$ GeV. (b) $d\sigma/dz$ (nb) as a function of z , at the center of mass energy $\sqrt{s} = 100$ GeV (HERA). The long dashed line is the leading order color singlet contribution. The sum of the leading order and $O(v^2)$ channels is the dot-dash line. The sum of the leading order and octet contribution is the dashed line. The solid line represents the total rate. The circles are the data from the H1 collaboration. (c) $d\sigma/dz$ (nb) at $\sqrt{s} = 14.7$ GeV. The lines represents contributions as described in (b). Data from EMC are shown with solid circles.











